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On the sensitivity of snow bidirectional reflectance to variations in grain characteristics

Petri M. Varsa[a] and Gladimir V.G. Baranoski[a]

[a]NPSG, D.R. Cheriton School of Computer Science, University of Waterloo, Waterloo, Ontario, Canada

ABSTRACT

Snow is a ubiquitous natural material that plays an important role in Earth's climatological system and energy resource budget. Its insular and reflective properties are key factors contributing to the radiation budget of the cryosphere. Due to its prevalence at extreme latitudes, the monitoring of snowpack quantities is often performed via remote observation. These data are acquired using either satellite readings or by fixing instruments to the underside of aircraft. When acquiring data remotely, it is important to account for the angular configuration of the source illumination and the location of the instrument relative to the surface since reflection is affected by the geometry of the observation. In other words, the bidirectional reflectance distribution function (BRDF) depends on the angle of incidence of the solar illumination and the angle of observation in addition to the wavelength of the incident light. It has been recognized that the granular properties of a snowpack markedly influence its BRDF. Unfortunately, works examining the effects of snow grain characteristics, such as size and facetness, on BRDF outputs are still scarce. Moreover, measured BRDF values from field studies presented in the literature are limited to specific target samples. This further hinders a more comprehensive understanding of the effects of changes in snow characterization parameters on the bidirectional reflectance of snowpack. The measured datasets often do not provide a detailed characterization of the target samples either, which also reduces their usefulness for elucidating these effects. To address these limitations and enhance the current understanding about the sensitivity of snow BRDF to variations in grain characteristics, we have conducted controlled experiments employing a first-principles *in silico* experimental framework supported by measured data. Our findings unveil the qualitative effects that snow granular properties have on bidirectional reflectance of snowpack, and highlight the importance of accounting for snow granular properties in remote sensing applications. In addition, our *in silico* experiments provide a high-fidelity assessment of snow BRDF with respect to key wavelengths particularly relevant for remote sensing applications. More broadly, our investigation demonstrates how remote observations of snow-covered terrains can be significantly improved by the correct incorporation of snow grain characteristics into the bidirectional reflectance models used to assess snowpack properties.

Keywords: snow, light transport, BRDF, faceted crystals

1. INTRODUCTION

Snow is an ubiquitous natural material primarily found at temperate latitudes and high elevations. Due to its reflective properties, it plays a key part in the regulation of Earth's climate.[1] It is also relevant for agriculture, providing source water for irrigation purposes at mid-latitudes.[2] Quantifying this water resource is a challenge since significant volumes are deposited in remote regions that are often difficult or time consuming to access. Consequently, remote observations are often required in order to assess the properties of snow deposits with regularity.

Data acquisition by remote systems requires that the geometry of the observation be taken into account. This is due to the fact that the directional component of reflectance, *i.e.*, the bidirectional reflectance distribution function (BRDF), of snow is not uniform. In addition to the angle of illumination and the angle of observation,

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Further author information: (Send correspondence to P.M.V.)
P.M.V.: E-mail: pmvarsa@uwaterloo.ca, Telephone: 1 519 888 4567
G.V.G.B.: E-mail: gvgbaran@cs.uwaterloo.ca, Telephone: 1 519 888 4567

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other factors may affect the BRDF response of a material. For example, the wavelength of the incident light. Material properties also play an important role.[3] Since the material properties of snow vary with time[4] and with environmental factors associated with the snowfall,[5] the BRDF values of fallen snow on the ground must take these variances in material properties into account. In order to properly assess the characteristics of the snowpack via remote observations, it is important to understand how BRDF responses are affected by these variances. For example, the grain size[6–9] and facetness[9] of snow are known to have an effect on the directional responses.

To date, a number of studies have been conducted, whereby the bidirectional reflectance of snow has been measured.[7–13] Each study contributed a noteworthy dataset to the literature. Whereas some of these datasets do include measurements of multiple samples that exhibit varying grain characteristics,[7,9,12] many datasets present measurements for just a single sample. As a consequence, it is difficult to isolate the effect that varying the values of a particular snow grain characteristic imparts on the BRDF.

To address these limitations and to add to the current knowledge about directional light reflectance from snowpacks, we propose that investigations be performed using an *in silico* (computational) investigation framework that takes into account the granular nature of the material. Accordingly, we carried out controlled experiments using measured spectral data references and snow characterization parameter values that are consistent with those provided in the literature. Our findings expand upon the trends previously observed to exist between snow grain facetness and bidirectional reflectance. They also provide an original assessment of the effect of facetness on bidirectional reflectance, which had previously only been reported anecdotally in the literature.

Our experiments were conducted using a first-principles hyperspectral light transport model known as SPLITSnow (*SP*ectral *LI*ght *T*ransport model for *Snow*).[14] The formulation of this model takes into account grain characteristics, such as the size distribution and facetness, in addition to sample characteristics such as sample density and snow depth.

The SPLITSnow model has been made available for online use[15] using a rapid model dissemination system.[16] Furthermore, the data used to conduct our experiments have also been made available for online access.[17] These resources have been published online in order to facilitate reproducibility of the results presented in this work. Using this system, individuals may execute goniometric reflectance experiments by specifying the morphological characterization values of the snow sample under test. Both numeric and graphical outputs are delivered to the user via electronic mail upon completion of the simulation.

Using this *in silico* approach, we investigate the bidirectional reflectance of two snow samples. The first sample consists of rounded grains and the second is composed of faceted grains, which are commonly referred to as faceted crystals.[5] In this work, we will use the terms *grain* and *crystal* interchangeably. The two (virtual) samples are irradiated using two different sources of illumination. The first illumination source is in the visible domain and is known to be highly reflective for snow. The second illumination source is in the infrared domain and is known to undergo a significant amount of absorption within snowpack while still exhibiting a complex pattern of bidirectional reflectance. To our knowledge, these trends have not been previously identified in the literature in such a manner as to isolate the effect of individual snow grain characteristics.

The remainder of this work is organized as follows. Section 2 presents a brief overview of bidirectional reflectance concepts. Section 3 outlines the *in silico* approach employed in our investigations. Section 4 presents the results of our experiments and discuses the implications of our findings. Finally, we present our concluding remarks and avenues for future research in Section 5.

2. PHYSICAL BACKGROUND

In this section, for clarity purposes, we provide an overview of radiometric terms employed in this work. It has been noted[18] that there has been a lack of consistency in radiometric terminology in the literature. For this reason, this paper will follow the standardization set out previously by other researchers.[18,19]

Fig. 1 provides a reference diagram for light incidence and collection geometry taken into account by the definitions presented in this section. The polar angle is measured away from the zenith and is denoted by θ. The symbol ϕ is used to represent the azimuthal angle, which increases counter-clockwise in the plane of the sample. The subscripts i and r indicate the incident and reflected directions, respectively.

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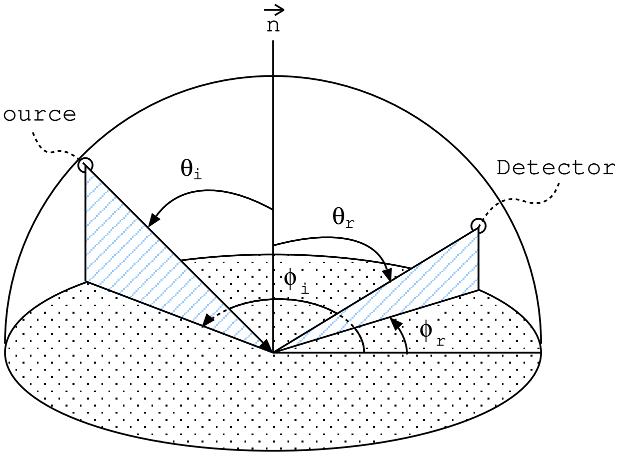

Figure 1. Definition of angular measure for incident, i, and reflected, r, directions. The polar angle, θ, increases away from the zenith (represented by the normal vector, $\vec{n}$), while the relative azimuth angle, ϕ, increases counter-clockwise in the plane of the sample.

The BSSDF (bidirectional surface-scattering distribution function) encapsulates the geometric reflectance (and transmittance) properties of a material. The proportion of reflected (or transmitted) radiance to incident flux (power) is specified for each incoming and outgoing direction and position on the material. If the scattering function does not depend upon the location of exitance, and the area of irradiance is relatively large, then only the directional components can be considered, yielding the BSDF (bidirectional scattering distribution function).[19] If only the reflected portion of the BSDF is of concern, it can be expressed in terms of the BRDF (bidirectional reflectance distribution function). The BRDF is thus defined as the ratio of reflected radiance to the incident irradiance.

There are three modes of electromagnetic radiation that have previously been defined.[19] They are *directional*, *conical* and *hemispherical*. Each mode must be considered for both incoming and reflected components. For example, the function relating the incident light from one direction to its contribution to the reflected light in another direction is directional-directional, otherwise called bidirectional. In the field or laboratory, this value is not directly measurable.[19] Light emitters and detectors utilized in field studies or in the laboratory always have a finite area, and thus are conical by design. However, with a simulation, it is possible to have directional incidence since the irradiance (the incident radiant flux (power) per unit area[19]) is specified using only two values: the point on the sample and the direction from the source of illumination. Hence, simulations are not restricted to area light sources. Accordingly, the following combinations are relevant to this discussion: directional-conical and conical-conical (biconical).

The BRDF is defined as the ratio of reflected radiance, $L_r(\theta_i, \phi_i; \theta_r, \phi_r; \lambda)$, to the incident irradiance, $E_i(\theta_i, \phi_i; \lambda)$:

$$f_r(\theta_i, \phi_i; \theta_r, \phi_r; \lambda) = \frac{\mathrm{d}L_r(\theta_i, \phi_i; \theta_r, \phi_r; \lambda)}{\mathrm{d}E_i(\theta_i, \phi_i; \lambda)} \qquad [\mathrm{sr}^{-1}] \qquad (1)$$

where (θ, ϕ) represents the directions in spherical coordinates and λ is the wavelength of light.

It is also valuable to have a clear picture about the physical constraints of a measurement apparatus. A source of illumination has a finite area by construction, which is directed at the sample to be measured. Similarly, a detector has a collection surface perpendicular to a particular direction of the upper hemisphere. Thus, a laboratory device used to measure reflectance is biconical in design. This method of measurement usually results in reflectance factor data.[18] For each geometric configuration, this is a ratio of reflected radiant flux from a sample to that reflected by an ideal, diffuse (Lambertian), standard surface. The biconical reflectance factor is given by:

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$$R(\theta_i, \phi_i, \omega_i; \theta_r, \phi_r, \omega_r) = \frac{\pi}{\Omega_i \Omega_r} \int_{\omega_i} \int_{\omega_r} f_r(\theta_i, \phi_i; \theta_r, \phi_r; \lambda)\ \mathrm{d}\Omega_r\, \mathrm{d}\Omega_i \tag{2}$$

where ω represents a solid angle and Ω represents a projected solid angle on the material surface.

3. METHODOLOGY

We performed *in silico* experiments that computed bidirectional reflectance of light through natural snow samples. The samples were specified by selecting distinct values for the characterization parameters, which affect the impinging rays of light. Bidirectional experiments were performed for each virtual sample to obtain our results. We preceded these experiments by conducting a baseline directional-hemispherical reflectance experiment and compared simulated values with measured values provided in the literature by Dumont *et al.*[9] This comparison was used to asses the suitability of the selected characterization datasets that were employed in the computation of the bidirectional reflectance plots.

We remark that, although the measured reflectance data values obtained by Dumont *et al.*[9] were sufficient to reproduce the hyperspectral signature of the snow sample that was measured, an insufficient level of detail was presented by way of sample characterization. For example, only an approximate grain size was given. Although we selected sample characterization parameter values that were based on details provided in their report,[9] we also made use of sample characterization parameter values that fall within physically valid ranges for snow[5] when optimizing the match between measured and simulated values.

The baseline measured reflectance curves obtained from the literature[9] are presented in Fig. 2 along side values obtained from our simulations. To compute the simulated reflectance curve, snow grains were generated using a uniform distribution with grain sizes in the range of 300–750 µm. The density of the sample was set to 0.275 $\mathrm{g\,cm^{-3}}$. Facetness is a unitless parameter that is used to model the rough details of a grain's surface. It allows for crystalline features to be added to the snow grains by way of microfacets.[20, 21] This characterization parameter takes values in the range of 0–1, with 1 being the the most faceted. A facetness of 0.3 was used for the baseline comparison. A 5% water saturation was used and the snow temperature was set to -1°C. The simulated snow grains are shaped as prolate spheroids. The shape distribution[22] employed a mean of 0.798 and a standard deviation of 0.064. The simulated sample depth was set to 12 cm to match the measured sample depth and the angle of incidence was set to 0° to match the experimental setup of the measured sample.

As it can be observed in the plot depicted in Fig. 2, the simulated values are in close agreement with the measured values. The root mean square error is less than 0.02, which is suitable for remote sensing applications.[23] We, therefore, applied the same sample characterization values used to specify this sample throughout our investigations. For the remainder of this work, we will refer to this characterization dataset as the representative sample.

The first-principles model, SPLITSnow,[14] used in our simulations employs an algorithmic formulation that is implemented using Monte Carlo methods[24] and ray optics concepts.[25] An impinging ray of electromagnetic radiation penetrates the surface of each sample where it interacts with the constituent snow crystals, making use of ray optics to interact with the stochastically generated particles. The particles are generated as needed by making stochastic use of the sample characterization parameter values. The impinging ray path is updated in this manner until it is either reflected, transmitted or absorbed. Upon exitance, the location and direction of the ray is recorded using a virtual goniophotometer.[26] We remark that the experiments conducted in this work do not depend upon the exitance location of each ray. Therefore, these locations are discarded, which yields the BSDF. In particular, our results are focused upon bidirectional reflectance and only the BRDF is considered.

As mentioned above, our experiments examine the effect of facetness on the BRDF. Faceted crystals are identified by sharp edges and corners. This is in contrast to grains that exhibit rounding at the edges and corners. Microfacets[20, 21] are employed in the formulation of the snow grains to achieve this purpose in the SPLITSnow framework. Thus, facetness is a unitless parameter ranging from zero to one, where zero indicates a perfectly smooth grain. In practice, the facetness is specified as a distribution of values to be selected from stochastically.

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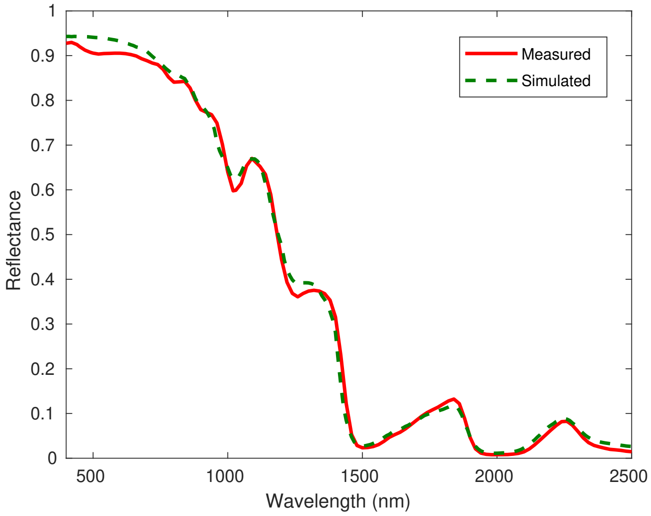

Figure 2. Comparison of measured[9] (red solid line) and simulated[14] (green dashed line) reflectance curves.

We performed our experiments using two separate wavelengths of light. More specifically, we selected 600 nm for our first set of experiments since snow is highly reflective in the visible domain at this wavelength. The second wavelength that we chose for experimentation was 1800 nm. This near infrared wavelength is much less reflective than 600 nm. It was selected since it exhibits an appreciable amount of reflectance and it is known to have reflectance features that differ in their directional component.[13]

4. RESULTS AND DISCUSSION

In this section, we initially present the results of our experiments that make use of a virtual 600 nm light source on the two samples. We then show the results of our experiments in which the samples are irradiated using an 1800 nm light source. Finally, we compare the outcomes of both experiments and discuss their implications for remote sensing applications.

Throughout this section, the characterization parameter values for the samples were all kept the same as those used for the representative sample introduced in Section 3. Only the independent variable under test was adjusted to suit the experimental conditions.

The plots in Fig. 3 present the results of our experiments using a 600 nm light source. Fig. 3(a) is a three-dimensional (3D) plot of the directional response for an illumination source impinging upon the material at an angle of 60° away from the zenith. The surface plot represents the magnitude of the directional reflectance in the upper hemisphere over the sample. The sample in Fig. 3(a) has an average facetness of 0.1. Similarly, Fig. 3(b) represents the same sample and illumination source. However, in Fig. 3(b), the mean facetness has been set to 0.9. Fig. 3(c) presents slices of the 3D plots that have been taken through the principal plane, which is the plane that contains both the normal to the surface and the direction of incidence.

We remark that snow is highly reflective at 600 nm (see Fig. 2). Yet even at this wavelength, the variation between the unfaceted and faceted samples is noticeable. The variation is not extensive, which is consistent with observations performed in field studies.[13] Nevertheless, a forward-scattering peak can be observed for the unfaceted (rounded) crystals, whereas the peak is reduced for faceted crystals. Although there have been observations reported in the literature whereby facets have been shown to affect bidirectional reflectance,[9,12] specific details of this effect are scarce. It is worth noting that, in field studies, it may be difficult to isolate the effect of facetness from the effect of grain size. This difficulty can be attributed to methodological constraints hindering the variation of only a single characterization trait of the measured samples.

The results of our second experiment are presented in Fig. 4. Similar to Fig. 3, Fig. 4(a) presents BRDF values for rounded crystals (mean facetness is 0.1), Fig. 4(b) presents BRDF values for highly faceted crystals (mean facetness is 0.9) and Fig. 4(c) presents a slice through the principle plane of the two 3D surfaces. Our

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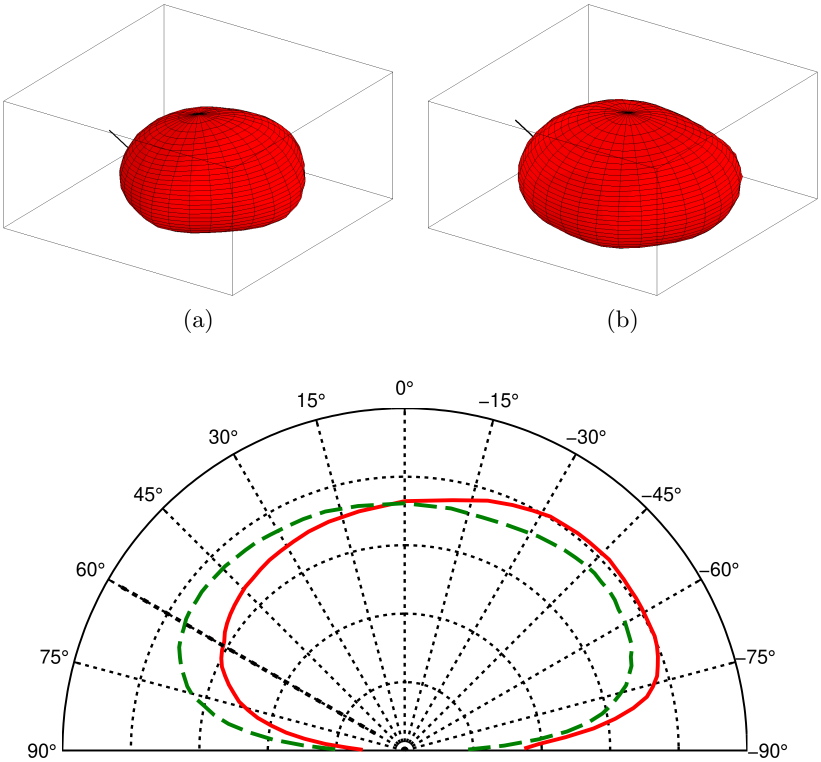

Figure 3. BRDF plots computed for the representative sample. A wavelength of 600 nm and an angle of incidence of 60° were employed. A facetness of 0.1 was employed to produce (a) whereas (b) was computed using a facetness of 0.9. The solid black line in (a) and (b) indicates the angle of incidence. Slices (through the principal plane) of these two surfaces are presented in (c). The red solid curve is a slice through (a) and the green dashed curve is a slice through (b). In (c), a black dotted line indicates the direction of incidence.

results indicate a markedly different response between rounded and faceted crystals. The sample characterized by rounded crystals demonstrated a greater forward-scattering peak, whereas the sample consisting of faceted crystals exhibited both forward- and backward-scattering peaks. These observations are consistent with the measured values presented by Painter and Dozier[12] where they observe a local backward-scattering peak in fine, faceted snow crystals. In addition to this, it has been observed by Aoki *et al.*[11] that bidirectional reflectance is known to have more directional variation in the near infrared. This phenomenon is also reproduced by our experiments, and it can be observed by comparing Figs. 3 and 4.

As shown in Fig. 2, spectral reflectance at 600 nm is significantly greater than at 1800 nm. To further illustrate this aspect, Fig. 5 presents a comparison of the directional profiles for faceted crystals (mean facetness is 0.9) for both 600 nm and 1800 nm depicted at the same scale. The red solid curve was produced using the same data that was used to produce the green dashed curve in Fig. 3(c), and the green dashed curve was produced using the same data as the green dashed curve in Fig. 4(c). We remark that the same sample was used to produce both curves, and that only the wavelength of the illumination source was altered.

It has been demonstrated[11,13,27] that there is an increase in the variance of directional reflectance (sometimes referred to as anisotropy) for wavelengths of light that are less reflective in the near infrared domain, when these are compared to wavelengths exhibiting greater total reflectance in the visible spectrum. This aspect has been attributed to an increase of the effect of single-scattering.[12] The association between grain size and directional reflectance has also received attention in the literature.[6–9,12,13] Although it has often been observed that an increase in grain size effects an increase in the variation of directional reflectance,[7,12] it has also been noted that this is wavelength dependent.[8]

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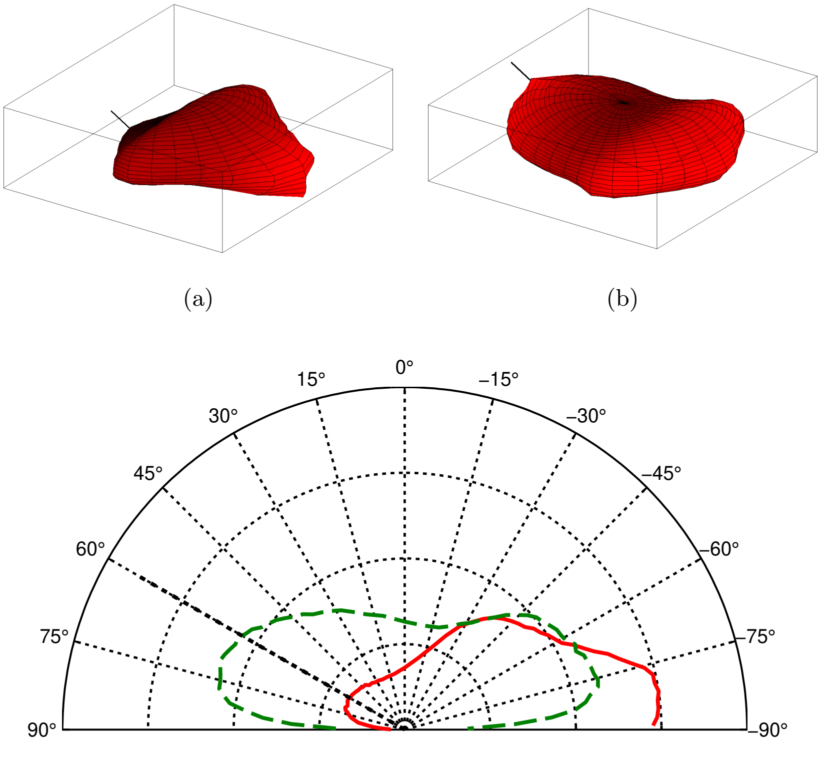

(a)

(b)

(c)

Figure 4. BRDF plots computed for the representative sample. A wavelength of 1800 nm and an angle of incidence of 60° were employed. A facetness of 0.1 was employed to produce (a) whereas (b) was computed using a facetness of 0.9. The solid black line in (a) and (b) indicates the angle of incidence. Slices (through the principal plane) of these two surfaces are presented in (c). The red solid curve is a slice through (a) and the green dashed curve is a slice through (b). In (c), a black dotted line indicates the direction of incidence.

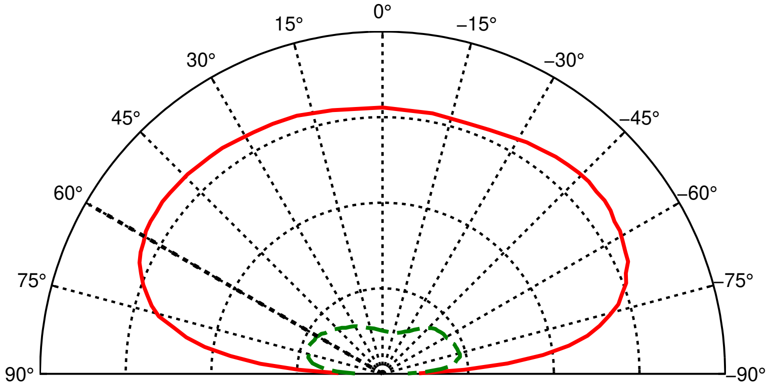

Figure 5. BRDF plots (through the principal plane) computed for the representative sample. An incident angle of 60° and a facetness were of 0.9 were employed to produce the plots. Two wavelengths namely 600 nm (red solid line) and 1800 nm (green dashed line) were employed to allow for a comparison to be made regarding relative brightness. A black dotted line indicates the direction of incidence.

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The morphological changes observed throughout the aging process of snow affect more than a single characterization trait, such as grain size, at a time.[28] Other characteristics, such as facetness are also affected by morphological changes. This imposes limitations on studies conducted in the field and complicates the analysis of their results.[9] By employing a first-principles *in silico* framework, we have been able to keep grain size constant and examine the effect of facetness on the BRDF of snow. Our experiments indicate that, at 1800 nm, snow crystals with low facetness exhibit marked variation throughout their directional response curves. In particular, directional reflectance in the forward direction is greater. Although less pronounced, this phenomenon has also been reproduced in our experiments considering an incident light at 600 nm.

In the case of natural snowpacks that have been precipitated as crystals, it is understood that morphological processes[28] alter the crystals so that they become more rounded (less faceted) over the course of several days. The same morphological processes also often contribute to the growth of the crystals, making the average crystal size increase with time. Due to the aforementioned limitations of field studies, it may be challenging to perform an *in situ* quantification of the effect of individual characterization value changes. In particular, as noted above, an increase in directional variation is often attributed to an increase in grain size. Our *in silico* experimental results indicate that grains size is not the only characteristic that affects the bidirectional reflectance.

Efforts have been made to estimate the grain size of snow using observations made in the near infrared domain.[29,30] As smaller new-fallen faceted crystals morphologically change into larger less faceted crystals over time, the effect on the BRDF in the infrared domain is non-trivial and is affected by changes due to both characterization traits. The increased size of the crystals tends to increase the variance of the directional reflectance.[11,13,27] However, our findings indicate that the decrease in facetness also augments the BRDF by increasing the amount of forward scattering. Therefore, attempts to assess snow grain size via remote observations could be improved by incorporating the faceted nature of the snow grains into these models.

5. CONCLUSION AND FUTURE WORK

In this work, we made use of a first-principles light transport model to conduct *in silico* experiments involving the bidirectional reflectance of snow. Given the abundance of snow and its relevance to climate and fresh water reserves, regular assessments of this natural resource are important to human welfare. Unfortunately, much of this natural material is precipitated at difficult or costly to access regions, thus requiring the use of remote observations for monitoring purposes. Comprehension of the non-uniform bidirectional reflectance distribution function for this material is essential for assessing snowpack properties on the ground. Not only are geometric and optical properties relevant, but material properties also affect directional reflectance. The effect of some of these material properties, such as grain size, has been examined in field and laboratory studies. Our work has shown that snow grain facetness is also an important characteristic, whose inclusion in investigation frameworks will only aid in obtaining more insightful knowledge and reliable data about this complex natural material.

In the future, we intend to extend this work by examining the BRDF for more wavelengths of interest. In particular, the 1030 nm band[30] and the 860 nm and 1640 nm bands[31] have been employed in snow grain size retrieval estimates. Examining a broader portion of the spectrum will aid in such applications. We also intend to investigate the effect that other snow sample characterization parameters have on the BRDF of snow. For example, snow grain shape (spherical vs. elongated), water content, and density may all have an impact on the spatial distribution of light propagated by snow. For some characteristics, rather than altering the qualitative form of the BRDF, a scaling of the response values may be detected. For such cases, an *in silico* investigation would ideally suit the task of determining the relationship between the characterization trait and the effect on the response (linear, quadratic, *etc.*). Finally, the transmittance of radiation through thin layers of snow has an effect on the radiation budget of frozen lakes.[32] It might also be interesting to account for the effects of the bidirectional transmittance distribution function in such environments.

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REFERENCES

[1] Barry, R. G., "The cryosphere and climate change," in [*Detecting the Climatic Effects of Increasing Carbon Dioxide*], MacCracken, M. C. and Luther, F. M. ., eds., (DOE/ER-0235), ch. 6, 109–148, US Dept. of Energy Washington, DC, Washington, DC (1985).

[2] Barnett, T., Adam, J., and Lettenmaier, D., "Potential impacts of a warming climate on water availability in snow-dominated regions," *Nature* **438**(7066), 303 (2005).

[3] Bohren, C. F. and Huffman, D. R., [*Absorption and scattering of light by small particles*], John Wiley & Sons, Toronto, Canada (1983).

[4] Colbeck, S. C., "Snow-crystal growth with varying surface temperatures and radiation penetration," *J. Glaciol.* **35**(119), 23–29 (1989).

[5] Fierz, C., Armstrong, R. L., Durand, Y., Etchevers, P., Greene, E., McClung, D. M., Nishimura, K., Satyawali, P. K., and Sokratov, S., "The international classification for seasonal snow on the ground," Tech. Rep. 83, UNESCO-IHP, Paris, France (2009).

[6] Warren, S. G. and Wiscombe, W. J., "A model for the spectral albedo of snow. II: Snow containing atmospheric aerosols," *J. Atmos. Sci.* **37**(12), 2734–2745 (1980).

[7] Kuhn, M., "Bidirectional reflectance of polar and alpine snow surfaces," *Ann. Glaciol.* **6**, 164–167 (1985).

[8] Warren, S. G., Brandt, R. E., and Hinton, P. O., "Effect of surface roughness on bidirectional reflectance of Antarctic snow," *J. Geoph. Res.-Planet.* **103**(E11), 25789–25807 (1998).

[9] Dumont, M., Brissaud, O., Picard, G., Schmitt, B., Gallet, J.-C., and Arnaud, Y., "High-accuracy measurements of snow bidirectional reflectance distribution function at visible and NIR wavelengths – Comparison with modelling results," *Atmos. Chem. Phys.* **10**(5), 2507–2520 (2010).

[10] Kuhn, M., "Spectroscopic studies at McMurdo, South Pole and Siple stations during the austral summer 1977-78," *Antarct. J. U.S.* **13**, 178–179 (1978).

[11] Aoki, T., Aoki, T., Fukabori, M., Hachikubo, A., Tachibana, Y., and Nishio, F., "Effects of snow physical parameters on spectral albedo and bidirectional reflectance of snow surface," *J. Geoph. Res.-Atmos.* **105**(D8), 10219–10236 (2000).

[12] Painter, T. H. and Dozier, J., "Measurements of the hemispherical-directional reflectance of snow at fine spectral and angular resolution," *J. Geoph. Res.-Atmos.* **109**(D18) (2004).

[13] Hudson, S. R., Warren, S. G., Brandt, R. E., Grenfell, T. C., and Six, D., "Spectral bidirectional reflectance of Antarctic snow: Measurements and parameterization," *J. Geoph. Res.-Atmos.* **111**(D18) (2006).

[14] Varsa, P. M., Baranoski, G. V. G., and Kimmel, B. W., "SPLITSnow: A spectral light transport model for snow," *Remote Sens. Environ.* **255**, 112272:1–20 (2021).

[15] Natural Phenomena Simulation Group, *Run SPLITSnow Online (Goniometric Mode)* (2020). http://www.npsg.uwaterloo.ca/models/splitsnow-gonio.php.

[16] Baranoski, G. V. G., Dimson, T., Chen, T. F., Kimmel, B. W., Yim, D., and Miranda, E., "Rapid dissemination of light transport models on the web," *IEEE Comput. Graph.* **32**(3), 10–15 (2012).

[17] Natural Phenomena Simulation Group, *Snow Data* (2020). http://www.npsg.uwaterloo.ca/data/snow.php.

[18] Schaepman-Strub, G., Schaepman, M. E., Painter, T. H., Dangel, S., and Martonchik, J. V., "Reflectance quantities in optical remote sensing—Definitions and case studies," *Remote Sens. Environ.* **103**(1), 27–42 (2006).

[19] Nicodemus, F., Richmond, J., Hsia, J., Ginsberg, I., and Limperis, T., [*Geometrical considerations and nomenclature for reflectance*], US Department of Commerce, National Bureau of Standards, Washington, D.C., USA (1977).

[20] Torrance, K. E. and Sparrow, E. M., "Theory for off-specular reflection from roughened surfaces," *J. Opt. Soc. Am.* **57**(9), 1105–1114 (1967).

[21] Cook, R. L. and Torrance, K. E., "A reflectance model for computer graphics," *ACM T. Graphic.* **1**(1), 7–24 (1982).

[22] Vepraskas, M. and Cassel, D., "Sphericity and roundness of sand in coastal plain soils and relationships with soil physical properties 1," *Soil Sci. Soc. Am. J.* **51**(5), 1108–1112 (1987).

[23] Jacquemoud, S., Ustin, S., Verdebout, J., Schmuck, G., Andreoli, G., and Hosgood, B., "Estimating leaf biochemistry using PROSPECT leaf optical properties model," *Remote Sens. Environ.* **56**, 194–202 (1996).

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[24] Hammersley, J. M. and Handscomb, D. C., [*Monte Carlo Methods*], Chapman and Hall, London, UK (1964).

[25] Born, M. and Wolf, E., [*Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*], Cambridge University Press, Cambridge, UK, 7 ed. (1999).

[26] Krishnaswamy, A., Baranoski, G. V. G., and Rokne, J. G., "Improving the reliability/cost ratio of goniophotometric comparisons," *J. Graph. Tools* **9**(3), 1–20 (2004).

[27] Dozier, J. and Painter, T. H., "Multispectral and hyperspectral remote sensing of alpine snow properties," *Annu. Rev. Earth Pl. Sc.* **32**, 465–494 (2004).

[28] Colbeck, S. C., "An overview of seasonal snow metamorphism," *Rev. Geophys.* **20**(1), 45–61 (1982).

[29] Nolin, A. W. and Dozier, J., "Estimating snow grain size using AVIRIS data," *Remote Sens. Environ.* **44**(2-3), 231–238 (1993).

[30] Nolin, A. W. and Dozier, J., "A hyperspectral method for remotely sensing the grain size of snow," *Remote Sens. Environ.* **74**(2), 207–216 (2000).

[31] Hong, G., "Estimating effective snow grain size using normalized channel ratios of MODIS 0.86 and 1.64 micron bands," in [*Geoscience and Remote Sensing Symposium (IGARSS)*], 2956–2959, IEEE (2020).

[32] Petrov, M. P., Terzhevik, A. Y., Palshin, N. I., Zdorovennov, R. E., and Zdorovennova, G. E., "Absorption of solar radiation by snow-and-ice cover of lakes," *Water Resour.* **32**(5), 496–504 (2005).

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