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Visually Assessing Maize Leaves: From Spectral Sampling to High-Fidelity Color Reproduction

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ABSTRACT

Maize (*Zea mays L.*, corn) crops are extensively used in food and biofuel production worldwide. A number of protocols have been proposed to use leaf color as an indicator of the health status of maize plants. Color perception is a complex process, however. The correct interpretation of its outcomes depends on several aspects. Accordingly, a variety of spectral vegetation indices have also been proposed to monitor the development of these plants. These indices usually require a number of spectral reflectance and transmittance samples taken from selected specimens using specialized sensors. Since these radiometric quantities do not depend neither on the spectra of the light sources nor on the physiological characteristics of the human visual system, these indices are not subject to color perception issues. The visual feedback provided by the chromatic attributes of plant leaves, on the other hand, can enable a broader assessment of the net effect of several environmental factors affecting an entire maize crop. Also, these attributes can be obtained using spectral reflectance and transmittance samples already employed in the computation of the aforementioned indices. These aspects indicate the potential benefits of the combined use of vegetation indices and leaf chromatic attributes in the monitoring of maize crops. Ideally, one would like to employ a number of spectral samples that would maximize the color fidelity to sensor costs ratio. In this paper, we address this practical trade-off. More specifically, using hyperspectral reflectance and transmittance data for maize specimens, we performed colorimetric experiments to obtain a lower bound for the number of spectral reflectance and transmittance samples sufficient to achieve a high degree of fidelity in the reproduction of maize leaves' colors under distinct illumination conditions.

Keywords: leaf, maize, corn, reflectance, transmittance, color perception, precision agriculture, remote sensing.

1. INTRODUCTION

Maize (*Zea mays L.*, corn) plants are among the most important cultivated species on the planet. To obtain an ecologically sustainable increase in the yield of maize crops, it's essential to monitor the health status of these plants in a timely and reliable fashion. Since the chromatic attributes of a leaf depend on the morphological characteristics and optical properties of its tissues, it can provide useful visual cues about biophysical phenomena that may have an impact on the plant's physiology. Not surprisingly, in the last years, a number of protocols have been proposed to use the maize leaves' chromatic attributes as indicators of such phenomena affecting their appearance.¹⁻⁶

A material's perceived color depends on its spectral signatures (reflectance and transmittance), the spectral power distribution of the illuminant (light source) and the responses of the observer's photoreceptors.⁷ The intrinsic complexity of this process can make the correct interpretation of a material's chromatic attributes a challenging task. For instance, two illuminants with distinct spectral power distributions may produce the same perception of colour, a phenomenon known as illuminant metamerism, or the same illuminant may produce different color perceptions for different observers, a phenomenon known as observer metamerism.^{8,9}

Alternatively, a variety of spectral vegetation indices have also been proposed to monitor plant development.^{10,11} These indices usually require a number of spectral samples of a target specimen's reflectance and transmittance signatures. Since these signatures do not depend neither on illuminants' emission spectra nor on human photoreceptors' responses, the subsequent analysis of these indices is not hindered by color perception issues.

It is worth noting that vegetation indices are usually designed to be used in the detection and monitoring of specific factors affecting a plant's health status. The visual inspection of leaf chromatic attributes, on the other hand, can provide a broader, albeit not necessarily complete, qualitative assessment of the net effect of several of these factors that may be concomitantly affecting a plant physiology. Also, leaf chromatic attributes can be obtained using spectral samples already employed in the computation of vegetation indices. These aspects provide an indication of the potential benefits that could result from the combined use of vegetation indices and leaf chromatic attributes in the cost-effective monitoring of maize crops' development.

There are practical issues related to this strategy that need to be considered, however. First, the monitoring of effects elicited by different environmental factors may require the use of several vegetation indices, with each index employing as few spectral samples as possible to mitigate sensor costs. Second, to reproduce the color of a target leaf specimen with a high level of fidelity,¹² it may be necessary to use a certain number of spectral samples. Ideally, one would like to employ a number of samples that would maximize the color fidelity to sensor costs ratio.

Assuming that one does not have prior information about the specimens' health status, it would be appropriate to employ an unsupervised spectral sampling strategy¹³ in the calculation of vegetation indices and the reproduction of leaf chromatic attributes. In other words, the spectral samples would be obtained at regular intervals within the visible spectral domain. This leads to the question of how many spectral samples would likely be sufficient to achieve a high degree of fidelity in the reproduction of leaf chromatic attributes, particularly considering small angles of light incidence, which is often the case in related remote sensing and low-proximity imaging applications.

In this work, we address this question. More specifically, using spectral measured reflectance and transmittance data for maize specimens and well-established colorimetry procedures, we carried out computations to estimate a lower bound for the number of spectral samples that might be sufficient to achieve a high degree of fidelity in the reproduction of maize leaves' chromatic attributes. To increase the scope of our observations, we considered different standard illuminants, normally associated with indoor and outdoor scenarios, as well as distinct regimes of light propagation by selected maize specimens. Although there are several models of light and plant interactions^{14,15} that can enable us to obtain maize reflectance and transmittance data for a wide range of *in silico* experimental conditions (*e.g.*, different angles of incidence), we opted to use actual measured plant spectral data¹⁶ so that we can provide a model-independent baseline for future investigations in this area.

2. METHODOLOGY

The measured spectral data employed in our investigation was made available in the LOPEX database.¹⁶ The LOPEX project involved experiments performed on 120 leaf specimens representative of more than 50 species, with the specimens collected during periods of high phenological activity. These experiments included directional-hemispherical reflectance and transmittance measurements carried out considering an angle of incidence of 8° (with respect to the specimens' normal vector). The resulting LOPEX spectral data files employed in our investigation were obtained from two distinct maize specimens, henceforth referred to M1 and M2. More precisely, the directional-hemispherical reflectance and transmittance for specimen M1 correspond to LOPEX spectral files 141 and 142, respectively, while the directional-hemispherical reflectance and transmittance for specimen M2 correspond to LOPEX spectral files 537 and 538, respectively.

For certain applications, color images or hyperspectral signatures of maize leaves are obtained taking into account only their reflective behaviour, *i.e.*, with one of the leaf's (blade) surfaces placed over an opaque material to block light transmission.^{11,17} In others, this procedure is not performed and the obtained images correspond to the sampling of the light reflected and transmitted by the leaves. We have considered both image capture set-ups. Moreover, the images can be obtained indoors (*e.g.*, in a greenhouse^{3,5}) or outdoors (*e.g.*, in a crop field^{4,6}). To account for that, we considered a standard CIE A illuminant (gas-filled tungsten lamp operating at a color correlated temperature of 2856 K)^{7,8} for the former case, and a standard D65 illuminant (average daylight with correlated color temperature of 6504 K) for the latter.^{7,8}

It has been demonstrated that the corn leaves' reflectance follows a near-Lambertian behaviour, notably for small angles of light incidence.¹⁸ As pointed out by Nicodemus *et al.*,¹⁹ for materials characterized by a predominantly diffuse (Lambertian) behaviour, directional-hemispherical reflectance and hemispherical-directional

reflectance factor quantities are equal for a given direction.¹⁹ We took this equivalence into account in this work and assumed that directional-hemispherical and hemispherical-directional reflectance quantities can be used interchangeably in the visualization of leaf chromatic attributes. Similarly, considering that unifacial corn leaves are characterized by a structural symmetry²⁰ and their transmittance follows a near-Lambertian behaviour¹⁸ as well, we also extended this rationale to the selected specimens' transmittance.

We then used the specimens' measured reflectance curves to generate the chromatic attributes associated with their light reflective behaviour. More precisely, these attributes were computed through the convolution of the selected illuminants' spectral power distributions, the measured reflectance data and the broad spectral responses of the human photoreceptors.⁷ This last step was performed by using a standard CIE XYZ to sRGB color system conversion procedure²¹ and considering the standard CIE A and D65 illuminants. Subsequently, the resulting colors were used to render leaf swatches through the application of a grayscale texture. The same procedure was used to generate the specimens' chromatic attributes associated with the aggregated combination of their reflective and transmissive behaviour. The components of this procedure are presented in Fig. 1.

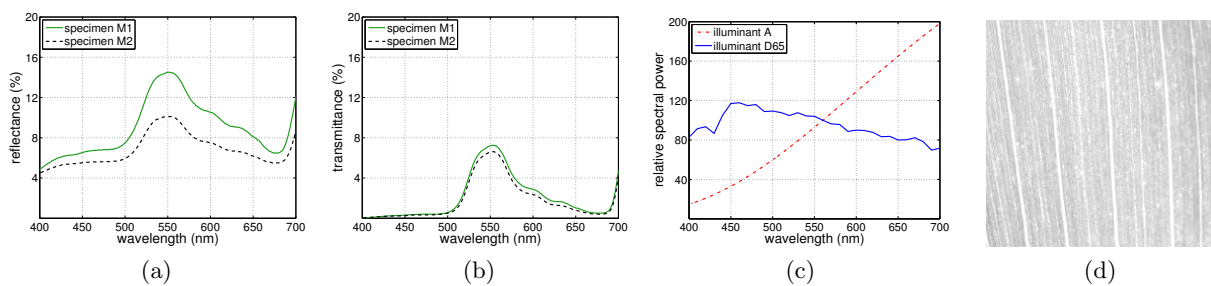


Figure 1: Components of the convolution procedure employed to generate the colors and swatches for the selected maize specimens. (a) Measured reflectance curves for specimens M1 and M2 (LOPEX spectral files 0141 and 0537,¹⁶ respectively). (b) Measured transmittance curves for specimens M1 and M2 (LOPEX spectral files 0142 and 0538,¹⁶ respectively). (c) Relative spectral power distributions associated with the CIE standard A (tungsten lamp) and D65 (daylight) illuminants.⁷ (d) Grayscale texture map (obtained from a photo of a maize leaf).

In short, our investigation involved a total of eight test cases: two specimens (M1 and M2), two light propagation behaviours (reflection only and aggregated reflection and transmission) and two illuminants (A and D65). For each test case, we computed the chromatic attributes associated with six distinct spectral sampling resolutions (Table 1) applied to the specimens reflectance and transmittance curves as well as to the illuminants' spectral power distributions. More explicitly, we considered six distinct values for the number of sampled wavelengths, denoted by N , used in the convolution procedure outline above. The lowest value ($N = 3$) corresponds to a standard nonuniform spectral sampling, and it was included for comparison purposes. It is associated with SMPTE (Society of Motion Picture and Television Engineers) monitor chromaticity coordinates often employed in the conversion of CIE XYZ values to the RGB color space.^{14,22} The intermediate values ($N = 5, 6, 7$ and 8) correspond to uniform (evenly-spaced) spectral sampling resolutions. Lastly, the largest value ($N = 301$) corresponds to the curves' full spectral resolution, and it was employed as a reference for fidelity assessments.

N	Spectral Intervals	Sampled Wavelengths
3	variable	465, 551 and 608 <i>nm</i> (monitor chromaticities)
5	75 <i>nm</i>	400, 475, 550, 625 and 700 <i>nm</i>
6	60 <i>nm</i>	400, 460, 520, 580, 640 and 700 <i>nm</i>
7	50 <i>nm</i>	400, 450, 500, 550, 600, 650 and 700 <i>nm</i>
8	37 <i>nm</i>	400, 437, 474, 511, 548, 585, 622, 659 and 696 <i>nm</i>
301	1 <i>nm</i>	all from 400 to 700 <i>nm</i> (full spectral resolution)

Table 1: Spectral resolutions (represented by the number (N) of sampled wavelengths) employed in our investigation.

Besides the visual inspection of the leaf swatches, we also employed a device-independent CIE-based metric to compare the obtained chromatic attributes. More specifically, we calculated the CIELAB differences between the colors (before their achromatic relative brightness modulation by the grayscale texture) associated with chosen spectral sampling resolutions ($N = 3, 5, 6, 7$ and 8) and the colors obtained considering the full spectral resolution ($N = 301$). These calculations were performed using the following formula:²³

$$\Delta E_{ab}^* = \sqrt{(d_L)^2 + d_a^2 + d_b^2}, \quad (1)$$

where d_L , d_a and d_b represent the differences $L_s^* - L_f^*$, $a_s^* - a_f^*$ and $b_s^* - b_f^*$, respectively, in which L^* , a^* and b^* correspond the CIELAB color space dimensions. These are calculated employing the chromatic attributes (colors) obtained considering the tested sparse spectral resolutions (indicated by the subscript s) and those attributes obtained considering the full spectral resolution (indicated by the subscript f). Again, we performed these calculations using standard formulas commonly employed in colorimetric studies²⁴ and considering the CIE A and D65 illuminants.⁷

3. RESULTS AND DISCUSSION

In Fig. 2, we present the swatches obtained considering the reflective behaviour of specimen M1. One can observe that the swatches generated using only 3 spectral samples fail to correctly reproduce the specimen's chromatic attributes obtained considering the full spectral resolution. Furthermore, as the number of samples is increased, the colors depicted in the swatches tend to match the colors of the reference swatches (generated using the full spectral resolution). Although the use of two different illuminants, A and D65, may alter the swatches' perceived colors, it does not affect the previously mentioned qualitative trends. These trends are also observed in the swatches presented in Fig. 3, which were obtained considering the aggregated reflective and transmissive behaviours of specimen M1. Since in this case more light is propagated through the specimen, the swatches appear brighter than those presented in Fig. 2.

In order to quantify the color variations observed in the swatches presented in Figs. 2 and 3, we computed the corresponding CIELAB ΔE_{ab}^* differences with respect to the reference swatches. The resulting values are provided in Table 2. It has been experimentally determined that the perceptibility threshold for CIELAB differences is 2.3,^{23,25,26} *i.e.*, chromatic variations associated with ΔE_{ab}^* below 2.3 are not considered discernible by human observers in general.²⁶ As it can be observed in Table 2, the ΔE_{ab}^* differences computed for the chromatic attributes obtained considering 7 evenly spaced samples are below the perceptibility threshold, albeit one may still be able to note some chromatic mismatches. By increasing the number of samples to 8, the resulting ΔE_{ab}^* differences become markedly below 2.3 and, for practical purposes, the colors become indistinguishable from those obtained considering the full spectral resolution.

When we repeated the procedures for the specimen M2, we observed the same qualitative trends reported for specimen M1. This can be verified in the swatches presented in Figs. 4 and 5. However, when we computed the corresponding CIELAB ΔE_{ab}^* differences, which are provided in Table 3, we noticed that, for some instances, 7 samples were no longer sufficient to obtain values below the 2.3 perceptibility threshold. Still, when we employed 8 evenly spaced samples, the resulting ΔE_{ab}^* differences become markedly below 2.3, and, again, the colors become indistinguishable from those obtained considering the full spectral resolution. This suggests that 8 samples may be considered a lower bound for the number of evenly spaced reflectance and transmittance values required to obtain a high-fidelity reproduction of the colors of healthy maize leaves.

Clearly, one can use different sampling schemes to obtain reflectance and transmittance values in the visible domain. Moreover, there is also a number of different illuminants and viewing/illumination geometries that can be employed to monitor the plants' development. Last, but not least, one should keep in mind the considerable variability among the characteristics and spectral signatures of maize specimens, and the fact that such a variability tends to increase as the plants' health status changes. Therefore, it may not be practical to obtain an exact lower bound for the number of spectral reflectance and transmittance samples. We remark, however, that the purpose of our investigation was to provide a basis for future experiments in this area. These could also involve other C_4 species, like sugarcane (*Saccharum officinarum*) plants, which not only share similar characteristics with maize plants (*e.g.*, unifacial leaves), but also have a similar importance for the food and biofuel production industries.²⁷

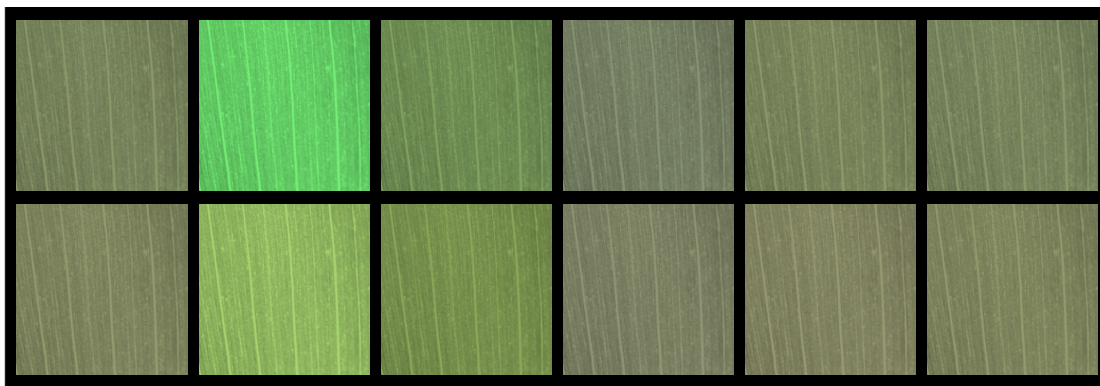


Figure 2: Swatches obtained considering the reflective behaviour of specimen M1, the relative spectral power distributions of the CIE standard A (top row) and D65 (bottom row) illuminants, and distinct spectral sampling resolutions. From left to right, 301 (full resolution), 3, 5, 6, 7 and 8 samples as indicated in Table 1.

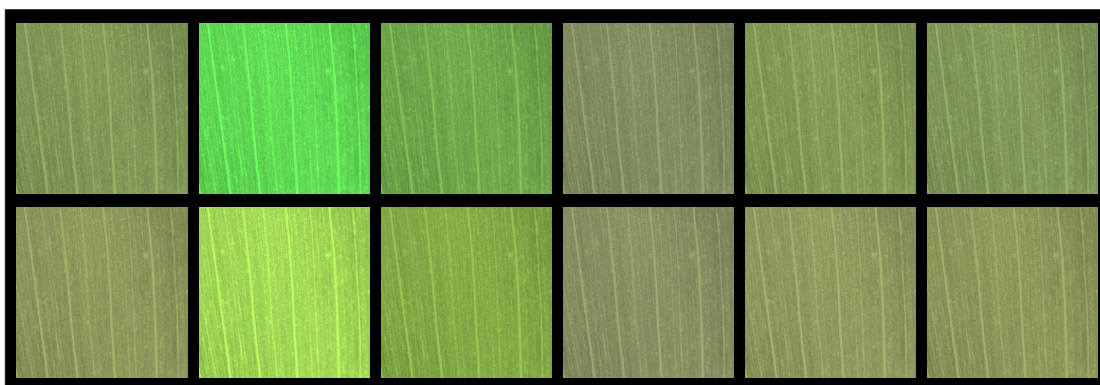


Figure 3: Swatches obtained considering the aggregated reflective and transmissive behaviour of specimen M1, the relative spectral power distributions of the CIE standard A (top row) and D65 (bottom row) illuminants, and distinct spectral sampling resolutions. From left to right, 301 (full resolution), 3, 5, 6, 7 and 8 samples as indicated in Table 1.

Illuminant	Reflective Behaviour					Aggregated Reflective and Transmissive Behaviour				
	$N = 3$	$N = 5$	$N = 6$	$N = 7$	$N = 8$	$N = 3$	$N = 5$	$N = 6$	$N = 7$	$N = 8$
A	46.7559	11.2375	6.4528	0.8461	0.8020	45.8230	17.1292	10.3191	1.9361	1.1052
D65	36.1066	19.7921	5.5571	2.0533	1.3456	46.1237	25.3037	9.8280	1.7179	0.8424

Table 2: CIELAB ΔE_{ab}^* differences computed for the M1 specimen's chromatic attributes obtained considering each tested sparse spectral resolution (N), the specimen's reflective behaviour (as depicted in the swatches presented in Fig. 2) and the specimen's aggregated reflective and transmissive behaviour (as depicted in the swatches presented in Fig 3). Values below the perceptibility threshold (2.3) for CIELAB chromatic differences are presented in boldface.

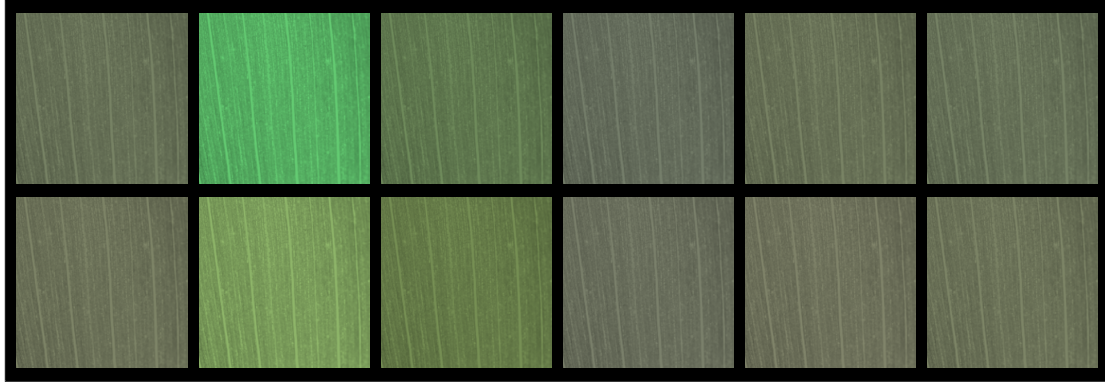


Figure 4: Swatches obtained considering the reflective behaviour of specimen M2, the relative spectral power distributions of the CIE standard A (top row) and D65 (bottom row) illuminants, and distinct spectral sampling resolutions. From left to right, 301 (full resolution), 3, 5, 6, 7 and 8 samples as indicated in Table 1.

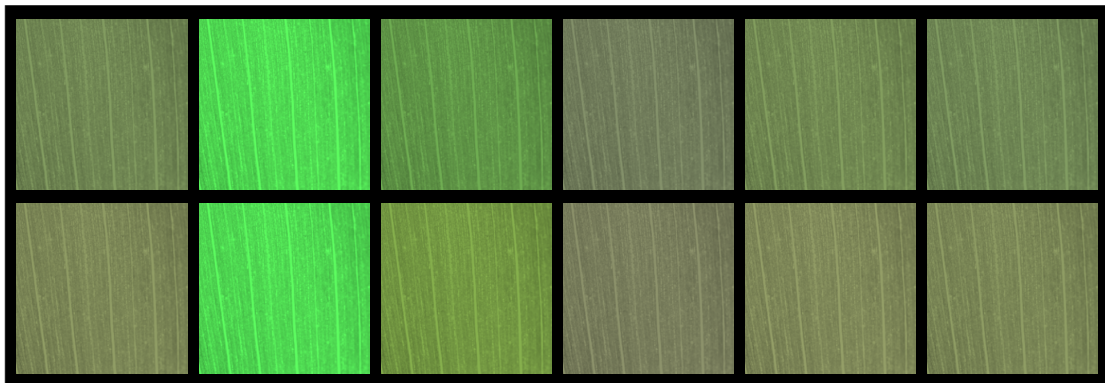


Figure 5: Swatches obtained considering the aggregated reflective and transmissive behaviour of specimen M2, the relative spectral power distributions of the CIE standard A (top row) and D65 (bottom row) illuminants, and distinct spectral sampling resolutions. From left to right, 301 (full resolution), 3, 5, 6, 7 and 8 samples as indicated in Table 1.

Illuminant	Reflective Behaviour					Reflective and Transmissive Behaviour				
	$N = 3$	$N = 5$	$N = 6$	$N = 7$	$N = 8$	$N = 3$	$N = 5$	$N = 6$	$N = 7$	$N = 8$
A	40.4986	9.7705	5.3314	0.6346	0.8277	53.5902	17.3101	9.9848	2.4778	0.7864
D65	31.1737	18.0523	4.6922	2.7911	0.7095	43.4365	24.4057	9.9055	1.4961	0.6757

Table 3: CIELAB ΔE_{ab}^* differences computed for the M2 specimen's chromatic attributes obtained considering each tested sparse spectral resolution (N), the specimen's reflective behaviour (as depicted in the swatches presented in Fig. 4) and the specimen's aggregated reflective and transmissive behaviour (as depicted in the swatches presented in Fig 5). Values below the perceptibility threshold (2.3) for CIELAB chromatic differences are presented in boldface.

4. CONCLUSION

Different approaches can be employed to monitor maize crops. No particular approach, either based on the calculation of vegetation indices or the analysis of foliar chromatic attributes, may be considered the “magic bullet” capable of providing the “best” feedback for all instances. The development of new technologies in this area will likely involve the implementation and possible combination of different methods, with the selection of a method for a particular application being based on the variables under scrutiny. Moreover, since most of the existing methods rely, directly or indirectly, on the interpretation of target specimens’ reflectance and transmittance signatures, it is only logical to search for cost-effective strategies for the sampling of this spectral information. This aspect has motivated the investigation presented in this paper. Such a search, however, can be viewed as a long term task since it involves several experimental dimensions. Nonetheless, the preliminary observations presented here suggest that it may be feasible to design multispectral sensors to measure reflectance and transmittance values at a relatively low number of wavelengths, and use this data to obtain high-fidelity reproductions of foliar chromatic attributes. This combined use of different monitoring methods could, in turn, potentially lead to more effective assessments of maize plants and other C4 species.

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